

Chapter 2. Relations and Functions

Question-1

Find x and y , if $(2x, x+y) = (6, 2)$.

Solution:

$$2x = 6$$

$$\therefore x = 3$$

$$x + y = 2$$

$$3 + y = 2$$

$$\therefore y = -1$$

Question-2

Find the domain of the following function : $f(x) = x|x|$

Solution:

The domain of the function $f(x) = x|x|$ is $\mathbb{R}$.

Question-3

Let $A = \{a, b, c\}$ and $B = \{p, q\}$. Find

(i) $A \times B$

(ii) $B \times A$

(iii) $A \times A$

(iv) $B \times B$

Solution:

(i) $A \times B = \{(a,p), (a,q), (b,p), (b,q), (c,p), (c,q)\}$

(ii) $B \times A = \{(p,a), (q,a), (p,b), (q,b), (p,c), (q,c)\}$

(iii) $A \times A = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c)\}$

(iv) $B \times B = \{(p,p), (p,q), (q,p), (q,q)\}$

Question-4

Find the domain of the following function : $f(x) = \frac{1}{\sqrt{x+|x|}}$

Solution:

$$x+|x| = 0 \text{ for } x < 0 \text{ or } x = 0.$$

$$\therefore \text{The domain of the function } f(x) = \frac{1}{\sqrt{x+|x|}} \text{ is } (0, \infty)$$

Question-5

Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{4, 5\}$. Verify that

$$(i) A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$(ii) A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Solution:

$$(i) \text{ L.H.S} = A \times (B \cap C) = \{1, 2, 3\} \times \{4\} = \{(1,4), (2,4), (3,4)\}$$

$$\text{R.H.S} = (A \times B) \cap (A \times C)$$

$$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\} \cap$$

$$\{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$$

$$= \{(1,4), (2,4), (3,4)\} \therefore A \times (B \cap C) = (A \times B) \cap (A \times C)$$

$$(ii) \text{ L.H.S} = A \times (B \cup C)$$

$$= \{1, 2, 3\} \times \{2, 3, 4, 5\}$$

$$= \{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5)\}$$

$$\text{R.H.S} = (A \times B) \cup (A \times C)$$

$$= \{(1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\} \cup$$

$$\{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$$

$$= \{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5), (3,2), (3,3), (3,4), (3,5)\}$$

$$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$$

Question-6

If R is the relation "less than" from $A = \{1, 2, 3, 4, 5\}$ to $B = \{1, 4, 5\}$, write down the set of ordered pairs corresponding to R . Find the inverse relation to R .

Solution:

$$R = \{(x, y) / x \in A, y \in B \text{ and } x < y\}$$

$$= \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,5)\}$$

$\therefore$ Inverse relation corresponds to the Cartesian product $\{(4,1), (5,1), (4,2), (5,2), (4,3), (5,3), (5,4)\}$ and corresponds to the relation 'greater than' from B to A .

Question-7

Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$.

Solution:

Let $x \in A \cap (B - C)$

$\Rightarrow x \in A$ and $x \in (B - C)$

$\Rightarrow x \in A$ and $\{x \in B \text{ and } x \notin C\}$

$\Rightarrow x \in A$ and $\{x \in B \text{ and } x \notin C\}$

$\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \notin C$

$\Rightarrow (A \cap B) - (A \cap C)$

Question-8

If $A = \{1, 2, 3\}$, $B = \{4\}$, $C = \{5\}$, then verify that

$$(i) A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$(ii) A \times (B - C) = (A \times B) - (A \times C)$$

Solution:

$$(i) A \times (B \cup C) = (A \times B) \cup (A \times C).$$

$$\begin{aligned} A \times (B \cup C) &= \{1, 2, 3\} \times \{4, 5\} \\ &= \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\} \end{aligned}$$

$$\begin{aligned} (A \times B) \cup (A \times C) &= \{(1, 4), (2, 4), (3, 4)\} \cup \{(1, 5), (2, 5), (3, 5)\} \\ &= \{(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)\} \end{aligned}$$

$$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$$

$$(ii) A \times (B - C) = (A \times B) - (A \times C)$$

$$\begin{aligned} A \times (B - C) &= \{1, 2, 3\} \times \{4\} \\ &= \{(1, 4), (2, 4), (3, 4)\} \end{aligned}$$

$$\begin{aligned} (A \times B) - (A \times C) &= \{(1, 4), (2, 4), (3, 4)\} - \{(1, 5), (2, 5), (3, 5)\} \\ &= \{(1, 4), (2, 4), (3, 4)\} \end{aligned}$$

Question-9

If R is the relation in $N \times N$ defined by $(a,b) R (c,d)$ if and only if $a + d = b + c$, show that R is an equivalence relation.

Solution:

Reflexive

$$\begin{aligned}(a,b) R (a,b) &\Leftrightarrow a + b = b + a \text{ for } a,b \in N \\ &\Leftrightarrow b + a = a + b \text{ (Transposing)} \\ &\Leftrightarrow (a,b) R (a,b) \text{ for } a,b \in N\end{aligned}$$

$$\therefore (a,b) R (a,b) \Leftrightarrow (a,b) R (a,b) \text{ for } a,b \in N$$

Symmetric

$$\begin{aligned}\text{If } (a,b) R (c,d) &\Leftrightarrow a + d = b + c \text{ for } a,b,c,d \in N \\ &\Leftrightarrow b + c = a + d \text{ (transposing)} \\ &\Leftrightarrow (c,d) R (a,b) \text{ for } a,b,c,d \in N\end{aligned}$$

$$\therefore (a,b) R (c,d) \Leftrightarrow (c,d) R (a,b) \text{ for } a,b,c,d \in N$$

Transitive

If $(a,b) R (c,d) \Leftrightarrow a + d = b + c$ for $a,b,c,d \in N$
and $(c,d) R (e,f) \Leftrightarrow c + f = d + e$ for $c,d,e,f \in N$

$$\begin{aligned}\text{then } (a,b) R (c,d) &\Leftrightarrow a + d = b + c \text{ for } a,b,c,d \in N \\ &\Leftrightarrow a + d + e + f = b + c + e + f \\ &\Leftrightarrow a + (d + e) + f = b + c + e + f \quad (\text{since } c + f = d + e) \\ &\Leftrightarrow a + c + f + f = b + c + e + f \\ &\Leftrightarrow a + f = b + e \\ &\Leftrightarrow (a,b) R (e,f) \text{ for } a,b,e,f \in N\end{aligned}$$

$$\therefore (a,b) R (c,d) \Leftrightarrow (a,b) R (e,f) \text{ for } a,b,c,d,e,f \in N$$

$\therefore$ the relation defined by $(a,b) R (c,d)$ if and only if $a + d = b + c$ is an equivalence relation.

Question-10

Find the domain of the following function : $f(x) = \frac{x}{x^2 - 3x + 2}$

Solution:

$x^2 - 3x + 2 = 0$ for $x = 2, 1$. $\therefore$ The domain of the function : $f(x) = \frac{x}{x^2 - 3x + 2}$ is $R - \{1, 2\}$.



Question-11

Let $A = \{1, 2, 3, 4\}$ and $S = \{(a, b) : a \in A, b \in A, a \text{ divides } b\}$. Write S explicitly.

Solution:

$$S = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

Question-12

Find the domain of the following function : $f(x) = e^{x+\sin x}$

Solution:

The domain of the function : $f(x) = e^{x+\sin x}$ is $\mathbb{R}$.

Question-13

Find the domain of the following function : $f(x) = \frac{x+7}{x^2-8x+4}$

Solution:

$$x^2 - 8x + 4 = 0 \text{ for } x = \frac{8 \pm \sqrt{64-16}}{2} = \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2} = 4 \pm 2\sqrt{3}$$

$$\text{The domain of the function: } f(x) = \frac{x+7}{x^2-8x+4} \text{ is } \mathbb{R} - \{4 \pm 2\sqrt{3}\}$$

Question-14

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write all subsets of $A \times B$.

Solution:

$$A \times B = \{(1,3), (1,4), (2,3), (2,4)\}.$$

$\therefore$ The subsets of $A \times B$ are $\phi, \{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\},$

$\{(1,3), (1,4)\}, \{(1,3), (2,3)\}, \{(1,3), (2,4)\}, \{(1,4), (2,3)\}, \{(1,4), (2,4)\}, \{(2,3), (2,4)\},$

$\{(1,3), (1,4), (2,3)\}, \{(1,3), (1,4), (2,4)\}, \{(1,3), (2,3), (2,4)\}, \{(1,4), (2,3), (2,4)\},$

$\{(1,3), (1,4), (2,3), (2,4)\}.$

Question-15

Find the domain of the following function : $f(x) = [x] + x$

Solution:

The domain of the function $f(x) = [x] + x$ is $\mathbb{R}$.



Question-16

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B, where x, y, z are distinct elements.

Solution:

$$A = \{x, y, z\} \text{ and } B = \{1, 2\}$$

Question-17

Find the domain of the following function : $f(x) = \frac{\sin^{-1} x}{x}$

Solution:

The domain of the function : $f(x) = \frac{\sin^{-1} x}{x}$ is $[-1, 1] - \{0\}$

Question-18

Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that $A \times C \subset B \times D$.

Solution:

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$B \times D = \{(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)\}$$

$$\therefore A \times C \subset B \times D.$$

Question-19

Find the range of each of the following function: $f(x) = |x - 3|$

Solution:

$f(x) = |x - 3|$ is positive for all values of x in R.

The range of the function $f(x) = |x - 3|$ is $(0, \infty)$.

Question-20

Let A be a non empty set such that $A \times B = A \times C$. show that $B = C$.

Solution:

Let $a \in A$. Since $B \neq \emptyset$, there exists $b \in B$. Now, $(a, b) \in A \times B = A \times C$ implies $b \in C$.

$\therefore$ every element in B is in C giving $B \subset C$. Similarly, $C \subset B$. Hence $B = C$.

Question-21

Find the range of the following function: $f(x) = 1 - |x - 2|$

Solution:

$$|x - 2| \geq 0 \Rightarrow 1 - |x - 2| \leq 1$$

The range of the function $f(x) = 1 - |x - 2|$ is $(-\infty, 1)$

Question-22

Find the range of the following function: $f(x) = \frac{|x - 4|}{x - 4}$

Solution:

$$f(x) = \frac{|x - 4|}{x - 4} = 1, \text{ if } x - 4 > 0$$

$$= -1, \text{ if } x - 4 < 0$$

$\therefore$ The range of the function $f(x) = \frac{|x - 4|}{x - 4}$ is $(-1, 1)$

Question-23

Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$. Let R be a relation from A to B defined by $R = \{(1, x), (1, z), (3, x), (4, y)\}$. Find the domain and range of R .

Solution:

Domain of $R = \{1, 3, 4\}$ and Range $R = \{x, y, z\} = B$

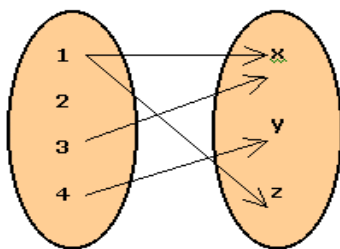
Question-24

Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$.

Let R be a relation from A to B defined by $R = \{(1, x), (1, z), (3, x), (4, y)\}$.

Draw the arrow diagram of relation R .

Solution:



Question-25

Find the range of the following function: $f(x) = \sqrt{16 - x^2}$

Solution:

The range of the function : $f(x) = \sqrt{16 - x^2}$ is $[0, 4]$.

Question-26

Find the range of the following function: $f(x) = \frac{1}{\sqrt{x-5}}$

Solution:

The range of the function is $(0, \infty)$

Question-27

In $\mathbb{N} \times \mathbb{N}$, show that the relation defined by $(a,b)R(c,d)$ if and only if $ad = bc$ is an equivalence relation.

Solution:

Reflexive

$$\begin{aligned}(a,b)R(a,b) &\hat{=} ab = ba \text{ for } a,b \in \mathbb{N} \\ &\hat{=} ba = ab \text{ (Transposing)} \\ &\hat{=} (a,b)R(a,b) \text{ for } a,b \in \mathbb{N}\end{aligned}$$

$$\therefore (a,b)R(a,b) \Leftrightarrow (a,b)R(a,b) \text{ for } a,b \in \mathbb{N}.$$

Symmetric

$$\begin{aligned}(a,b)R(c,d) &\hat{=} ad = bc \text{ } a,b,c,d \in \mathbb{N} \\ &\hat{=} bc = ad \text{ (Transposing)} \\ &\hat{=} (c,d)R(a,b) \text{ for } a,b,c,d \in \mathbb{N}\end{aligned}$$

$$\therefore (a,b)R(c,d) \Leftrightarrow (c,d)R(a,b) \text{ for } a,b,c,d \in \mathbb{N}$$

Transitive

$$\begin{aligned}\text{If } (a,b)R(c,d) &\Leftrightarrow ad = bc \text{ } a,b,c,d \in \mathbb{N} \\ \text{and } (c,d)R(e,f) &\Leftrightarrow cf = de \text{ } c,d,e,f \in \mathbb{N}\end{aligned}$$

$$\begin{aligned}\text{Then } (a,b)R(c,d) &\Leftrightarrow ad = bc \text{ } a,b,c,d \in \mathbb{N} \\ &\hat{=} adef = bcef \text{ (Multiplying both sides by } ef) \\ &\hat{=} adef = be(cf) \\ &\hat{=} adef = bede \text{ (Since } cf = de) \\ &\hat{=} af = be \\ &\hat{=} (a,b)R(e,f) \text{ } a,b,e,f \in \mathbb{N}\end{aligned}$$

$$\therefore (a,b)R(c,d) \hat{=} (a,b)R(e,f) \text{ } a,b,e,f \in \mathbb{N}$$

$\therefore$ the relation defined by $(a,b)R(c,d)$ if and only if $ad = bc$ is an equivalence relation.

Question-28

Find the domain and the range of the following function : $f(x) = \frac{1}{\sqrt{x - [x]}}$

Solution:

We know that $0 \leq x - [x] \leq 1$ for all $x \in \mathbb{R}$. Also, $x - [x] = 0$ for $x \in \mathbb{Z}$.

$f(x) = \frac{1}{\sqrt{x - [x]}}$ is defined if $x - [x] > 0$

i.e., $x \in \mathbb{R} - \mathbb{Z}$.

Hence the domain of the function is $\mathbb{R} - \mathbb{Z}$.

Question-29

Find the domain and the range of the following function : $f(x) = \frac{1}{\sqrt{4 + 3 \sin x}}$

Solution:

$$-1 \leq \sin x \leq 1 \Rightarrow -3 \leq 3 \sin x \leq 3$$

$$\text{i.e. } -1 \leq 4 + 3 \sin x \leq 7$$

$$\therefore \frac{1}{\sqrt{7}} \leq y \leq 1$$

The domain of the function is $\mathbb{R}$; Range : $\frac{1}{\sqrt{7}} \leq y \leq 1$

Question-30

Let R be the relation on $\mathbb{Z}$ defined by $a R b$ if and only if $a - b$ is an even integer. Find (i) R , (ii) domain R , (iii) range of R .

Solution:

(i) $R = \{(a, b) : a \text{ and } b \text{ are even integers}\} \cup \{(c, d) : c \text{ and } d \text{ are odd integers}\}$

(ii) Domain = $\mathbb{Z}$

(iii) Range = $\mathbb{Z}$

Question-31

Find the domain and the range of the following function : $f(x) = 1 - |x - 3|$

Solution:

The domain of the function is $\mathbb{R}$; Range : $(-\infty, 1)$

Question-32

Let R be the relation on $\mathbb{Z}$ defined by $R = \{(a, b) : a \in \mathbb{Z}, b \in \mathbb{Z}, a^2 = b^2\}$. Find (i) R , (ii) domain R , (iii) range of R .

Solution:

(i) $R = \{(a, a) : a \in \mathbb{Z}\} \cup \{(a, -a) : a \in \mathbb{Z}\}$

(ii) Domain = $\mathbb{Z}$

(iii) Range = $\mathbb{Z}$



Question-33

Find the domain and the range of the following function : $f(x) = x!$

Solution:

The domain of the function is $N \cup \{0\}$; Range : $\{n! : n = 0, 1, 2, \dots\}$

Question-34

Determine the domain and the range of the relation R defined by $R = \{(x+1, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

Solution:

Domain = $\{1, 2, 3, 4, 5, 6\}$, Range = $\{5, 6, 7, 8, 9, 10\}$

Question-35

Determine the domain and the range of the relation R, where $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$.

Solution:

Domain = $\{2, 3, 5, 7\}$, Range = $\{8, 27, 125, 343\}$

Question-36

Find the domain and the range of the following function : $f(x) = \sin^2(x^3) + \cos^2(x^3)$

Solution:

$$\sin^2(x^3) + \cos^2(x^3) = 1$$

$\therefore$ The domain of the function is R ; Range : $\{1\}$



Question-37

Is inclusion of a subset in another, i.e., $A \subseteq B$ if and only if $A \subset B$, in the context of a universal set, an equivalence relation in the class of subsets of the universal set? Justify your answer.

Solution:

Let U be the universal set. Let R be the relation 'is a subset of' or 'is included in' between the subsets of U .

Since every set is a subset of itself i.e., for every subset A in U , $A \subseteq A$ or $A R A$.

$\therefore R$ is reflexive.

Now let A and B be two subsets of U such that $A \subseteq B$, then it is not necessary that B must also be a subset of A .

$\therefore A R B$ need not imply $B R A$.

$\therefore R$ is not symmetric.

Hence R is not an equivalence relation.

Question-38

Find the domain and the range of the following function : $f(x) = \frac{x^2 - 9}{x - 3}$

Solution:

$$f(x) = \frac{x^2 - 9}{x - 3} = x + 3$$

The domain of the function is $\mathbb{R}$; Range : $\mathbb{R}$.

Question-39

Determine the domain and range of the following relations

- (i) $\{(1, 2), (1, 4), (1, 6), (1, 8)\}$
- (ii) $\{(x, y) : x \in \mathbb{N}, y \in \mathbb{N} \text{ and } x + y = 10\}$
- (iii) $\{(x, y) : x \in \mathbb{N}, x < 5, y = 3\}$
- (iv) $\{(x, y) : y = |x - 1|, x \in \mathbb{Z} \text{ and } |x| \leq 3\}$

Solution:

- (i) Domain = $\{1\}$, Range = $\{2, 4, 6, 8\}$
- (ii) Domain = $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, Range = $\{9, 8, 7, 6, 5, 4, 3, 2, 1\}$
- (iii) Domain = $\{1, 2, 3, 4\}$, Range = $\{3\}$
- (iv) Domain = $\{-3, -2, -1, 0, 1, 2, 3\}$, Range = $\{4, 3, 2, 1, 0\}$



Question-40

How many relations are possible from a set A of m elements to another set B of n elements? Why?

Solution:

Number of elements in A = m.

Number of elements in B = n

∴ Number of elements in $A \times B = mn$

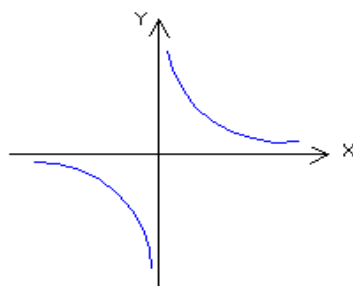
Number of subsets of $A \times B = 2^{mn}$

Since every subset of $A \times B$ is a relation from A to B therefore 2^{mn} relations are possible from A to B.

Question-41

Draw the graph of the following function: $f(x) = \frac{1}{x}, x \neq 0$

Solution:



Question-42

Let $A = \{1, 2\}$. List all the relations on A.

Solution:

$$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

The relations on A are $\phi, \{(1, 1)\}, \{(1, 2)\}, \{(2, 1)\}, \{(2, 2)\},$

$$\begin{aligned} &\{(1, 1), (1, 2)\}, \{(1, 1), (2, 1)\}, \{(1, 1), (2, 2)\}, \\ &\{(1, 2), (2, 1)\}, \{(1, 2), (2, 2)\}, \{(2, 1), (2, 2)\}, \\ &\{(1, 1), (1, 2), (2, 1)\}, \{(1, 1), (1, 2), (2, 2)\}, \\ &\{(1, 1), (2, 1), (2, 2)\}, \{(1, 2), (2, 1), (2, 2)\}, \\ &\{(1, 1), (1, 2), (2, 1), (2, 2)\} \end{aligned}$$

Question-43

Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A into B.

Solution:

$$n(A) = 3 \text{ and } n(B) = 2$$

$$\therefore n(A \times B) = 2 \times 3 = 6$$

$\therefore$ the number of relations from A into B are $2^6 = 64$.

Question-44

Which of the following relations are functions? If it is a function, determine its domain and range:

- (i) $\{(2, 1), (5, 1), (8, 1), (11, 1), (14, 1), (17, 1)\}$
- (ii) $\{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)\}$
- (iii) $\{(0, 0), (1, 1), (1, -1), (4, 2), (4, -2), (9, 3), (9, -3), (16, 4), (16, -4)\}$
- (iv) $\{(1, 2), (1, 3), (2, 5)\}$
- (v) $\{(2, 1), (3, 1), (5, 2)\}$
- (vi) $\{(1, 2), (2, 2), (3, 2)\}$

Solution:

- (i) Domain = $\{2, 5, 8, 11, 14, 17\}$, Range = $\{1\}$
- (ii) Domain = $\{2, 4, 6, 8, 10, 12, 14\}$, Range = $\{1, 2, 3, 4, 5, 6, 7\}$
- (iii) No, As there are four pairs of ordered pairs which have the same first element.
- (iv) No, As two ordered pairs which have the same first element.
- (v) Domain = $\{2, 3, 5\}$, Range = $\{1, 2\}$
- (vi) Domain = $\{1, 2, 3\}$, Range = $\{2\}$

Question-45

If $A = \{1, 2, 3\}$, $B = \{a, b\}$, find $A \times A$.

Solution:

$$A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Question-46

Find the domain and range of the following functions:

- (i) $\left\{ \left(x, \frac{x^2 - 1}{x - 1} \right) : x \in \mathbb{R}, x \neq 1 \right\}$
- (ii) $\{(x, -|x|) : x \in \mathbb{R}\}$

Solution:

- (i) Domain = $\mathbb{R} - \{1\}$, Range = $\mathbb{R} - \{2\}$
- (ii) Domain = $\mathbb{R}$, Range = $\{y : y \in \mathbb{R} \text{ and } y \leq 0\}$

Question-47

If $A = \{1, 2, 3\}$, $B = \{a, b\}$, find $A \times B$

Solution:

$$A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$$

Question-48

Find the domain and range of the following functions:

(i) $\{(x, \sqrt{9-x^2}) : x \in \mathbb{R}\}$

(ii) $\left\{\left(x, \frac{1}{1-x^2}\right) : x \in \mathbb{R}, x \neq \pm 1\right\}$

Solution:

(i) Domain = $\{x : x \in \mathbb{R} \text{ and } -3 \leq x \leq 3\}$, Range = $\{y : y \in \mathbb{R} \text{ and } -3 \leq y \leq 3\}$

(ii) Domain = $\mathbb{R} - \{1, -1\}$, Range = $\{y : y \in \mathbb{R}, y \neq 0, y < 0 \text{ and } y \geq 1\}$

Question-49

If $A = \{1, 2, 3\}$, $B = \{a, b\}$, find $B \times B$.

Solution:

$$B \times B = \{(a, a), (a, b), (b, a), (b, b)\}$$